Comparison of Three Geometric Parameterisation Methods and Their Effect on Aerodynamic Optimisation

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Summary

The choice of shape parameterisation enormously impact on the design space and optimal solution in the aerodynamic optimisation. Three parameterisation methods, PARSEC, the Class/Shape Function Transformation (CST) and MACROS Dimension Reduction (DR), which is a novel parameterisation method, are studied in this paper. Comparison studies of these methods are performed in terms of accuracy of inverse fitting and effect on constructing design space. The results show that MACROS DR has excellent capability of dimension reduction and significantly high accuracy of inverse fitting. The CST and PARSEC methods can provide higher flexibility than MACROS DR comparing their design space.

Keywords: *Shape Parameterisation, Parsec, Class/Shape Function Transformation, MACROS DR, Design Space*

1. Introduction

Aerodynamic shape optimisation plays more and more important role in aircraft design. Shape parameterisation methods enormously impact on the results of aerodynamic optimisation. In general, the current shape parameterisation methods used in aerodynamic optimisation could be classified into eight categories[9]: Basis vector, Domain element, Partial Differential Equation, Discrete (mesh point), Polynomial and Spline, Analytical, CAD-based and Free-form deformation. Samareh[9] has reviewed and compared these methods, and pointed out that successful parameterisation methods should have following properties: 1) compact on the number of design variables, 2) providing the high flexibility to cover the optimal solution in design space, 3) representing existing geometries with high accuracy, 4) producing smooth and realistic shape.

Few researchers have investigated the effect of different shape parameterisation methods on optimisation process. Sripawadkul[12] studied and compared five aerofoil parameterisation methods, Ferguson's curves, Hicks-Henne bump functions, B-Spline, PARSEC and Class/Shape function transformation method (CST), in terms of parsimony, completeness, orthogonality, flawlessness and intuitiveness. Five parameterisation methods were scored to assist to select the proper method respect to specific issue. Song and Keane[11] investigated effect of two parameterisation methods, orthogonal basis function and B-Spline, on inverse fitting the different aerofoils. The results showed the B-spline could provide higher accuracy than orthogonal basis function using high number of design variables. Castonguay[3] studied the effect of four parameterisation methods, mesh points, B-Spline, Hicks-Henne bump function and PARSEC, on inverse design and drag minimisation in 2D aerofoil. The results demonstrated the mesh points method provides the highest level of accuracy comparing to other methods, and PARSEC may be unable to provide high flexibility since it failed in inverse design case. Mousavi[8] performed the 2D aerofoil inverse design, 2D drag minimisation and 3D wing drag minimisation using mesh points, B-Spline and CST methods. It showed the mesh points method provided the best results in all test cases. The B-Spline and CST methods were able to provide the reasonable accuracy with low number of design variables. The CST was able to eliminate the shock wave using very low number of variables in drag minimisation case.

In this work, three parameterisation methods for 2D aerofoil, PARSEC, CST and MACROS DR, are studied and compared. MACROS DR[1,2,4] is a novel parameterisation method developed by DATADVANCE. The performance of inverse fitting of three methods is compared in terms of geometric error and pressure distribution error between original and approximated aerofoil. Their effect on aerodynamic optimisation was investigated by generating their design space.

2. Geometric Parameterisation and Benchmarking

2.1. The Description of Three Parameterisation Methods

Three geometric parameterisation methods for aerofoil are employed in this work. PARSEC as a specific parameterisation method for aerofoil has been widely used in aerodynamic optimisation. The CST has been becoming a popular parameterisation method since it is able to produce a wide range of shapes. MACROS DR[1,2] is a novel parameterisation method to reduce the dimension of design space.

2.1.1 PARSEC Method

The PARSEC parameterisation method is developed by Sobieczky[10]. The purpose of the method is to find a minimum number of variables to address the special aerodynamic or flow features. In this method, eleven intuitive parameters are employed to explicitly represent aerofoil as showed in Figure 1. They are the leading edge radius (R_{le}), upper crest position (X_{up}, Z_{up}), upper crest curvature (Z_{xxup}), lower crest position (X_{lo}, Z_{lo}), lower crest curvature (Z_{xxlo}), trailing edge position (Z_{te}), trailing thickness (ΔZ_{te}) and two trailing edge angles (α_{te} and β_{te}).

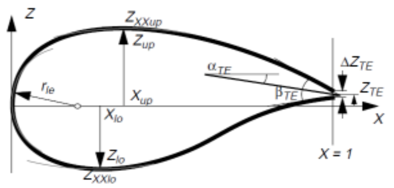

Figure 1 The PARSEC Parameterisation Method

Polynomials are employed to describe the upper and lower aerofoil shape:

$$Z = \sum_{n=1}^{6} a_n X^{n-\frac{1}{2}} \tag{1}$$

The coefficients a_n can be obtained solving the following linear system of equations:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ X_{TE}^{\frac{1}{2}} & X_{TE}^{\frac{3}{2}} & X_{TE}^{\frac{5}{2}} & X_{TE}^{\frac{7}{2}} & X_{TE}^{\frac{9}{2}} & X_{TE}^{\frac{11}{2}} \\ X_{UP}^{\frac{1}{2}} & X_{UP}^{\frac{3}{2}} & X_{UP}^{\frac{5}{2}} & X_{UP}^{\frac{7}{2}} & X_{UP}^{\frac{9}{2}} & X_{UP}^{\frac{11}{2}} \\ \frac{1}{2}X_{TE}^{-\frac{1}{2}} & \frac{3}{2}X_{TE}^{\frac{1}{2}} & \frac{5}{2}X_{TE}^{\frac{3}{2}} & \frac{7}{2}X_{TE}^{\frac{5}{2}} & \frac{9}{2}X_{TE}^{\frac{7}{2}} & \frac{11}{2}X_{TE}^{\frac{9}{2}} \\ \frac{1}{2}X_{UP}^{-\frac{1}{2}} & \frac{3}{2}X_{UP}^{\frac{1}{2}} & \frac{5}{2}X_{UP}^{\frac{3}{2}} & \frac{7}{2}X_{UP}^{\frac{5}{2}} & \frac{9}{2}X_{UP}^{\frac{7}{2}} & \frac{11}{2}X_{UP}^{\frac{9}{2}} \\ -\frac{1}{4}X_{TE}^{-\frac{3}{2}} & \frac{3}{4}X_{UP}^{-\frac{1}{2}} & \frac{15}{4}X_{UP}^{\frac{1}{2}} & \frac{35}{4}X_{UP}^{\frac{3}{2}} & \frac{53}{4}X_{UP}^{\frac{5}{2}} & \frac{99}{4}X_{UP}^{\frac{7}{2}} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} = \begin{bmatrix} \sqrt{2R_{le}} \\ Z_{TE} \\ Z_{UP} \\ \tan(\theta_{TE,UP}) \\ 0 \\ \left[\frac{\partial^2 Z}{\partial X^2}\right]_{X=X_{UP}} \end{bmatrix} \tag{3}$$

2.1.2 The Class/Shape Function Transformation Method (CST)

The CST method, proposed by Kulfan[7], is recently interesting to researchers. The purpose of this method is to develop a universal parameterisation for complex aircraft configuration. The CST for two-dimensional aerofoil can be written as follow:

$$\xi(\psi) = C_{N2}^{N1}(\psi) \cdot S(\psi) + \psi \cdot \Delta\xi_{te} \tag{5}$$

Where the $C_{N2}^{N1}(\psi)$ is the class function, $S(\psi)$ is the shape function and $\Delta\xi_{te}$ is the trailing edge thickness. N_1 and N_2 are the class parameters, and are set to 0.5 and 1.0 respectively for the aerofoil with the round nose and aft-end trailing edge. Bernstein polynomial is employed as the shape function to describe the detailed shape. Hence, the completed mathematic equation of the CST aerofoil could be written as follow:

$$\xi(\psi) = \psi^{0.5} \cdot (1-\psi)^{1.0} \cdot \sum_{i=0}^{n} [A_i \cdot \binom{n}{i} \cdot \psi^i \cdot (1-\psi)^{n-i}] + \psi \cdot \Delta\xi_{te} \tag{6}$$

2.1.3 MACROS DR

MACROS Dimension Reduction[1,2] (MACROS DR) is a general-purpose method for dimension reduction which also can be used for aerodynamic applications, in particular, for parameterisation of aerofoils. There exists special domain-specific extension of MACROS DR[4], elaborated exclusively for parameterisation of aerofoils, although it is not considered in this paper.

The procedure of MACROS DR is a particular case of iterative dimension reduction procedures on basis of non-linear framed orthogonal design manifolds[1,2], and can be considered as a nonlinear generalization of principal component analysis (nonlinear manifold approximating multidimensional description of considered objects is constructed). The procedure consists of two main steps:

1) Linear manifold is constructed using principal component analysis and then

2) Nonlinear deviation of multidimensional object description from this linear manifold is approximated by its nonlinear projection onto the several main principal components expanding linear manifold constructed on the previous step.

2.2. The Benchmarking

Representing the existing aerofoil with high accuracy is one of the requirements of successful parameterisation method. Therefore, the performance of inverse fitting of three parameterisation methods is assessed by measuring the difference between the original aerofoil and approximated aerofoil. The root mean square (RMS) error is used to assess this difference as Figure 2 and equation (7).

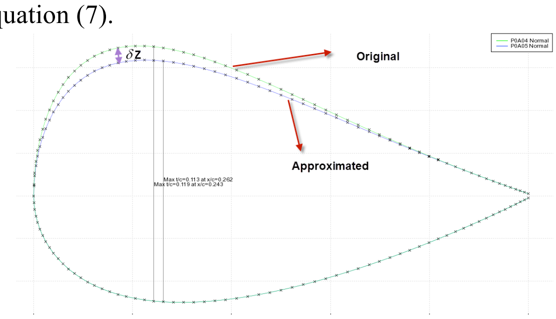

Figure 2 RMS of Geometric Fitting

$$RMSE = \sqrt{\sum_i \left(z_i - z_{i,approx}\right)^2} \tag{7}$$

Because the aerodynamic flow behaviouris highly sensitive to the surface curvature, it means a little perturbation of geometry could cause the large variation on pressure distribution. Therefore, it is not enough that only the geometric RMS is compared. The difference of pressure distribution between the original and approximated aerofoil is considered in this work. BVGK (Viscous Garabedian-Korn) which can predict the aerodynamic characteristic in subsonic free-stream with decent accuracy is employed as the CFD solver [5,6]. The same RMS criteria, however, is applied to conduct the pressure distribution of profile rather than the geometry, see Figure 3. In additional, the effect of incresing dimension of MACROS DR and the CST is studied.

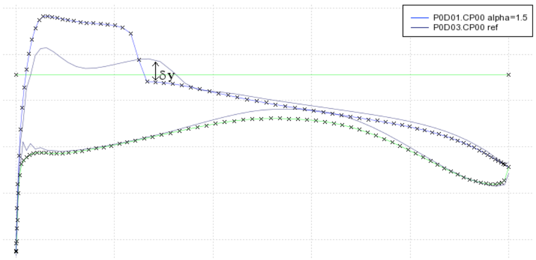

Figure 3 RMS of Pressure Distribution

Moreover, providing design space with high flexibility is another important requirement to shape parameterisation. In this work, the design space of three parameterisation methods are compared and investigated. In order to set up the design space and guarantee design space from different methods are comparable, the following processes are carried out.

1) A set of aerofoils are fitted using three methods
2) Obtaining bounds of parameters for each parameterisation method by finding the minimum and maximum value for each parameter.
3) Extending the bound 5% on minimum and maximum
4) Using an Optimal Latin Hypercube algorithm, which is one of design of experiment algorithm, to generate the sample points within the given bound.
5) Running the BVGK solver to every sample point, and the flow condition is set to $\alpha = 1.5$, $Re = 3.6 \times 10^6$, $M = 0.7$
6) Plotting the C_L and C_D for all points where the CFD solver worked to visualise the design space.

Once the design space was generated, the effect of different parameterisation methods could be investigated and compared by observing distribution of parametric aerofoils. In the meantime, amount of parametric aerofoils failed in BVGK solver is monitored to understand the effect of parameterisation on producing the realistic shape.

3. Results

3.1. Geometric Inverse Fitting

Figure 4 illustrates variants of the averaging geometric RMS obtained by three methods with increasing the dimension. For each dimension, 20 aerofoils are fitted to calculate averaging RMS. Because the number of design variables of PARSEC is fixed, the curve of PARSEC is plotted constantly with increase the dimension. It shows MACROS DR and the CST methods yield much higher accuracy than the PARSEC method. Both of MACROS DR and the CST methods could achieve below 2×10^{-3}. However, the number of design variables of MACROS DR is much less than the CST at same level of accuracy. It shows MACROS DR has the excellent capability of reduction of the design space. The dimension of design space could be compressed to only 4 dimensions, which is difficult to achieve using other methods.

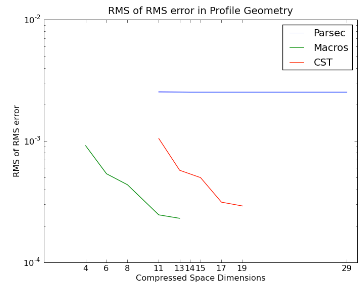

Figure 4 Comparison of Geometric RMS using three parameterisation methods

The tests of RMS of Cp distribution are performed and shown in Figure 5. As the poor ability of inversing fitting of PARSEC in geometrc side, the high deviation of Cp distribution is occured in the test cases of PARSEC. Similar to the geometric RMS, both MACROS DR and the CST methods reach reasonable level of accuary. MACROS DR can achieve low RMS with only 8 variables. Although the CST could achieve low geometric RMS with increasing high number of design variables, the RMS of pressure distribution would be turned to high. The reason is the high approximated oscillation is generated while the high number of design variables is used.

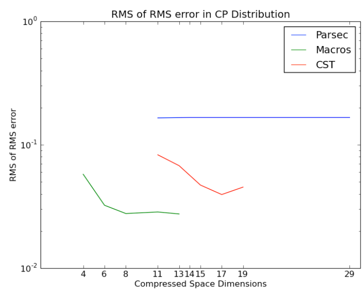

Figure 5 Comparison of RMS of Cp Distribution using three parameterisation methods

3.2. Comparison of Design Space

The design space of three parameterisation methods are then established and plotted as Figure 6-8. The dimensions of design space of three methods are all set to 11. The initial set of training aerofoils are highlighted in the design space as the blue points. Figure 6 illustrates the CST method provides the wide range of aerofoil through the design domain. And the CST parametric aerofoils are mainly distributed around the training set and low drag area. Figure 8 illustrates PARSEC parametric aerofoils are evenly distributed in design domain, and can reach the higher lift area comparing to the design space of CST. Figure 7 shows the design space of MACROS DR. It is distributed in smaller area and only around the training set.

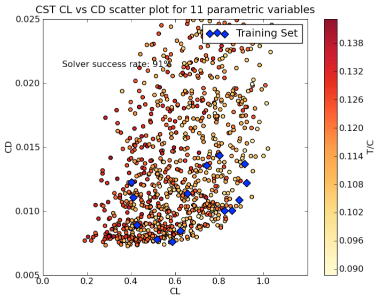

Figure 6 Design Space Generated using CST with 11 Design Variables

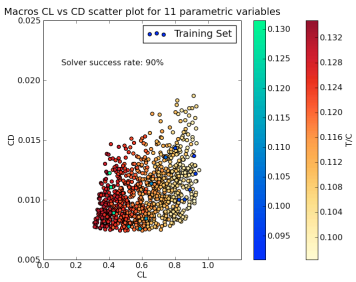

Figure 7 Design Space Generated using MACROS DR with 11 Design Variables

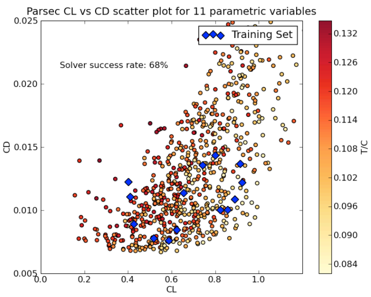

Figure 8 Design Space Generated using PARSEC with 11 Design Variables

The other data, solver success rate, should be discussed. The rate of the CST and MACROS DR are 91% and 90%, respectively. It means the most parametric aerofoils which are generated by the CST and MACROS DR are reasonable and realistic. However, the rate of PARSEC is only 68%, which means the high rate of unrealistic parametric aerofoils are generated in PARSEC.

4. Conclusion

Three parameterisation methods have been tested and compared. MACROS DR has excellent capability of inverse fitting to reach high level of accuracy on both geometric and pressure distribution comparing to other methods. The CST could achieve the same level of accuracy, however a larger number of design variables are needed. PARSEC with 11 parameters is not enough to provide high level of accuracy of inverse fitting. MACROS DR provides high ability of dimension reduction, which can represent aerofoils with only 4 parameters in some cases.

The design space of three methods are generated and compared as well. The results show PARSEC and the CST can provide a wide range of profiles, and MACROS DR provides a reasonable design space around the initial training set. It is a compromise to choose the higher flexibility design space and low dimension with high accuracy of inverse fitting while selecting the parameterisation.

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